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The Geometric Foundations of Matter and Spacetime: Rigidity, Visibility, and the Topology of Compactification

Foundations of the Emergent Universe of Mathematical Structure

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Abstract

This paper addresses three foundational questions within the Fracture–Berry–Tension (FBT) framework: (i) how symplectic rigidity serves as a unifying geometric principle underlying both quantum uncertainty and gravitational regularity; (ii) how visible and dark matter sectors are to be distinguished geometrically; and (iii) how black holes should be understood in a six-dimensional symplectic ontology.

We show that Gromov's non–squeezing theorem provides a common geometric explanation for the existence of a minimal action quantum and for the impossibility of true geometric singularities. Building on this, we classify matter sectors in terms of two geometric invariants of compactified surfaces

$$\Sigma^2 \hookrightarrow \mathcal{M}^6: \qquad A_\Sigma = \int_{\Sigma^2} \Omega, \qquad c_1^{\mathrm{ind}}(\Sigma).$$

Visible matter requires both nonzero symplectic area and nontrivial induced gauge topology, together with admissible observable phase locking. Dark sectors arise either when the induced gauge topology is trivial or when coherent observable readout fails despite gravitational activity. Measure arguments suggest that dark compactifications are generic, whereas visible compactifications are structurally selective. Black holes are reinterpreted as regions where regular observable projection fails, rendering the interior observationally inaccessible while the underlying six-dimensional geometry remains regular.

Keywords: Symplectic Rigidity, Gromov Non-Squeezing, Compactification Invariants, Induced Chern Class, Visible and Dark Sectors, Phase Locking, Black Hole as Projection Failure, Event Horizon as Visibility Boundary, FractureBerryTension Framework.

Contents

1 Introduction

The Fracture–Berry–Tension (FBT) framework develops a geometric account of physical emergence based on a six-dimensional symplectic ontology. Within this programme, the present paper addresses three foundational questions.

(i) Symplectic rigidity as a unifying principle. Why does quantum theory exhibit a minimal action scale, and why do gravitational singularities resist interpretation as genuine geometric objects? We argue that both issues are traced back to one common source: symplectic rigidity in the sense of Gromov's non–squeezing theorem. This gives a unified geometric foundation for quantum uncertainty and gravitational regularity.

(ii) The geometric definition of visible and dark matter. What distinguishes visible matter from dark matter at the level of compactified geometry? We propose that the distinction is controlled primarily by two invariants of compactified surfaces

$$\Sigma^2 \hookrightarrow \mathcal{M}^6 : \qquad A_\Sigma = \int_{\Sigma^2} \Omega, \qquad c_1^{\mathrm{ind}}(\Sigma).$$

The first controls gravitational activity, while the second controls gauge admissibility in the emergent four-dimensional sector. Visibility further requires an admissible observable phase locking condition, so that gauge-active compactifications are not merely present in principle but dynamically readable in the effective projection.

(iii) Black holes as projection failure. What is a black hole in a six-dimensional symplectic universe? We argue that black holes are not singularities of the underlying geometry, but regions where regular observable projection to four-dimensional spacetime fails. This makes the event horizon a boundary of observability rather than a boundary of existence.

The paper is organized as follows. Section 2 develops symplectic rigidity as a unifying principle. Section 3 presents the geometric classification of visible and dark sectors. Section 4 interprets black holes as extreme cases of projection failure. Section 5 summarizes the conceptual consequences.

Throughout, we assume the basic FBT setup: a six-dimensional symplectic manifold

$$(\mathcal{M}^6, \Omega)$$

admitting an effective 4+2 organization

$$\Sigma^2 \hookrightarrow \mathcal{M}^6 \to \mathcal{M}^4_{\mathrm{obs}},$$

where Σ^2 is the compact phase fibre and $\mathcal{M}^4_{\mathrm{obs}}$ is emergent spacetime.

2 Symplectic Rigidity as a Unifying Principle

The arguments below rely on the geometric identification of the Planck constant h as the minimal symplectic period on admissible compactification cycles. A rigorous derivation of this identification, based on prequantisation theory and the structural discreteness of the FBT geometry, is provided in Appendix C.

2.1 Gromov's Non-Squeezing Theorem

Theorem Σ-2.1 (Gromov 1985)**.** *Let $B^{2n}(R)$ be the standard symplectic ball of radius R in*

$$(\mathbb{R}^{2n}, \omega_0 = \sum dp_i \wedge dq^i),$$

and let

$$Z^{2n}(r) = B^2(r) \times \mathbb{R}^{2n-2}$$

be the symplectic cylinder of radius r. There exists a symplectic embedding

$$\varphi : B^{2n}(R) \hookrightarrow Z^{2n}(r)$$

if and only if $R \le r$.

In physical terms, one cannot compress the symplectic area of a two-dimensional projection below its original value while preserving the symplectic structure. This yields a symplectic capacity $c(\cdot)$, monotone under symplectomorphisms, which controls the minimal transverse area of symplectic embeddings.

Definition Σ-2.2 (Symplectic capacity)**.** A symplectic capacity assigns to each subset $U \subset (\mathcal{M}^{2n}, \Omega)$ a number $c(U) \in [0, \infty]$ such that:

(i) $c(U) \le c(V)$ if there exists a symplectic embedding $U \hookrightarrow V$;

(ii) $c(B^{2n}(R)) = c(Z^{2n}(R)) = \pi R^2$;

(iii) $c(\lambda U) = \lambda^2 c(U)$ for scaling.

2.2 From Rigidity to Quantum Uncertainty

Within the FBT framework, the fundamental symplectic manifold $\mathcal{M}^6$ is not directly observable. Physical measurements occur in the four-dimensional projection $\mathcal{M}^4_{\text{obs}}$, obtained after compactification along the phase sector.

Proposition Σ-2.3 (Geometric origin of uncertainty)**.** *Let γ be a loop in $\mathcal{M}^4_{\text{obs}}$ that bounds a two-chain σ whose lift to $\mathcal{M}^6$ has nonzero symplectic area*

$$A = \int_{\tilde{\sigma}} \Omega.$$

Under projection, the uncertainty in simultaneously measuring conjugate variables along γ satisfies

$$\Delta q \, \Delta p \ge \frac{A}{2\pi}. \tag{1}$$

Proof sketch. Locally, Darboux coordinates give

$$\Omega = dp \wedge dq + \cdots .$$

The projected area in the (q, p)-plane is bounded below by the lifted symplectic area through non–squeezing. Identifying the minimal nonzero admissible symplectic period with h then yields the uncertainty bound. □

Corollary Σ-2.4 (Planck constant as minimal symplectic period)**.** *The Planck constant h is identified with the minimal nonzero symplectic period on admissible compactification cycles:*

$$h = \min \left\{ \int_{\Sigma^2} \Omega > 0 \ : \ [\Sigma^2] \in \mathcal{H}_2^{\mathrm{adm}}(\mathcal{M}^6, \mathbb{Z}) \right\}. \tag{2}$$

Remark Σ-2.5. The role of non–squeezing is to protect the stability of this minimal period. The integrality and existence of the minimal generator are derived in Appendix C from quantum-phase consistency and prequantisation.

2.3 From Rigidity to Singularity Censorship

Theorem Σ-2.6 (No true singularity in six-dimensional ontology)**.** *Let $\mathcal{R} \subset \mathcal{M}^6$ be a region supporting compactification surfaces Σ^2 with symplectic area*

$$A_\Sigma \geq h > 0.$$

Then there exists no sequence of admissible symplectic deformations that compresses the symplectic capacity of $\mathcal{R}$ to zero. In particular, no point in $\mathcal{R}$ can be a geometric singularity in the six-dimensional description.

Proof. Any neighbourhood of a candidate singular point contains a transverse symplectic two-disk whose area is bounded below by h. By non–squeezing, its symplectic capacity cannot be driven to zero. □

Corollary Σ-2.7 (Four-dimensional singularities as projection artefacts)**.** *Apparent singularities in the effective four-dimensional spacetime $\mathcal{M}^4_{\mathrm{obs}}$ arise where the projection*

$$\pi : \mathcal{M}^6 \to \mathcal{M}^4_{\mathrm{obs}}$$

ceases to be regular. The six-dimensional geometry remains regular and finite-capacity throughout.

2.4 Why Local Geometry Cannot Be Ontological

A decisive conceptual consequence follows from the absence of local symplectic invariants. By Darboux's theorem, any symplectic manifold is locally symplectomorphic to the standard

$$(\mathbb{R}^{2n}, \omega_0).$$

There is no intrinsic local notion of symplectic curvature or local symplectic shape.

Principle Σ-2.8 (Non-locality of symplectic ontology)**.** The fundamental ontology of the FBT framework admits no local geometric degrees of freedom at the symplectic level. All physically significant structure is encoded in global and topological invariants such as symplectic capacities, period lattices, and the topology of compactification cycles.

This has three immediate consequences:

- quantum non-locality is structurally expected;
- spacetime metric structure is emergent rather than fundamental;
- global response channels classify modes of observable emergence rather than local forces.

3 The Geometric Definition of Dark Matter

3.1 Two Fundamental Invariants of Compactified Surfaces

In the FBT framework, matter sectors arise from compactified two-dimensional symplectic surfaces embedded in $\mathcal{M}^6$:

$$\Sigma^2 \hookrightarrow (\mathcal{M}^6, \Omega). \tag{3}$$

Each such surface carries two independent classes of geometric data.

Definition Σ-3.1 (Symplectic area)**.** The symplectic area of Σ^2 is

$$A_\Sigma := \int_{\Sigma^2} \Omega. \tag{4}$$

This quantity controls the contribution of the compactification to spacetime curvature in the emergent four-dimensional sector. Through the gravitational readout channel, A_Σ manifests as inertial and gravitational mass.

Definition Σ-3.2 (Induced first Chern class)**.** Let

$$\pi : \mathcal{M}^6 \to \mathcal{M}^4_{\mathrm{obs}}$$

be the projection to emergent spacetime. The embedding

$$\Sigma^2 \hookrightarrow \mathcal{M}^6,$$

together with the Berry curvature on the compact phase fibre, induces a $U(1)$ gauge bundle on $\pi(\Sigma^2) \subset \mathcal{M}^4_{\mathrm{obs}}$. The induced first Chern class

$$c_1^{\mathrm{ind}}(\Sigma) \in H^2(\mathcal{M}^4_{\mathrm{obs}}, \mathbb{Z}) \tag{5}$$

is the Chern class of this bundle. It measures the topological obstruction to a global section and controls gauge admissibility.

Remark Σ-3.3 (Intrinsic vs. induced)**.** The notation $c_1^{\mathrm{ind}}(\Sigma)$ emphasizes that this is not the intrinsic Chern class of Σ^2 as an abstract two-manifold, but the class induced by embedding and projection.

3.2 Visible Matter: Gauge Activity and Observable Phase Locking

Definition Σ-3.4 (Visible matter surface)**.** A compactified surface Σ^2_{vis} corresponds to visible matter if it satisfies:

(i) **Gravitational activity:**

$$A_\Sigma \neq 0;$$

(ii) **Topological gauge activity:**

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0;$$

(iii) **Observable phase locking:** the internal phase structure of Σ^2 enters an admissible observable readout regime, so that the induced gauge degrees of freedom are coherently projected into the temporal order of the effective four-dimensional sector.

The third condition is not topological but observational. A compactification may satisfy the structural conditions for gauge activity while remaining observationally suppressed if its internal phase data fail to enter a stable, temporally readable effective projection.

Theorem Σ-3.5 (Gauge field induction). *If*

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0,$$

then upon projection to $\mathcal{M}^4_{\mathrm{obs}}$ *there exists a nontrivial* $U(1)$ *gauge connection* A *whose curvature satisfies*

$$F = dA \propto c_1^{\mathrm{ind}}(\Sigma). \tag{6}$$

The associated electric charge is quantized:

$$q \propto \int_{\pi(\Sigma^2)} c_1^{\mathrm{ind}}(\Sigma) \in \mathbb{Z}. \tag{7}$$

Proof sketch. By the Chern–Weil homomorphism, the cohomology class of

$$\frac{1}{2\pi} F$$

is identified with c_1^{ind}. Hence nonzero induced first Chern class forces nontrivial gauge curvature, and integrality gives charge quantization. □

3.3 Dark Matter: Geometric Decoupling

Definition Σ-3.6 (Dark matter surface). A compactified surface Σ^2_{dark} corresponds to a dark sector if it satisfies:

(i) **Gravitational activity:**

$$A_\Sigma \neq 0;$$

(ii) **Visibility failure:** either

$$c_1^{\mathrm{ind}}(\Sigma) = 0$$

(topological gauge inactivity), or coherent observable phase locking fails, so that the sector remains dynamically unreadable in the effective four-dimensional projection despite gravitational activity.

Dark matter is therefore not introduced here as a new elementary particle species, but as a geometric sector that:

- contributes to the energy–momentum structure through its symplectic area and hence gravitates;
- either fails to induce gauge activity at the topological level, or fails to enter an admissible observable phase-locking regime;
- remains invisible to detectors whose operation presupposes gauge-active and temporally readable matter sectors.

Proposition Σ-3.7 (Geometric decoupling). *Darkness is not a consequence of weak coupling, but of geometric decoupling: gauge interactions require nontrivial induced topology, and observability requires coherent phase locking. The absence of either removes visibility at the structural level.*

3.4 Why Dark Matter Dominates by Measure

Theorem Σ-3.8 (Measure dominance of dark compactifications)**.** *In the space of admissible embeddings*

$$\Sigma^2 \hookrightarrow \mathcal{M}^6$$

satisfying nonzero area, stability, and symplectic discreteness, the subset for which both

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0$$

and admissible observable phase locking are realised is structurally nongeneric. By contrast, compactifications that fail either gauge activity or coherent observable readout constitute the generic case.

Proof sketch. The condition

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0$$

already requires a nontrivial and selective twisting of the induced gauge structure. Observable phase locking imposes an additional coherence constraint on the phase data of the compactification. Since each condition is nongeneric, their simultaneous satisfaction is structurally selective, whereas dark compactifications dominate by measure. □

Corollary Σ-3.9 (Dark sectors as default, visible sectors as selective)**.** *Visible matter occupies a structurally selective subset of admissible compactifications, while dark sectors represent the generic case.*

3.5 Observable Phase Locking as a Visibility Condition

The discussion above shows that visibility requires three logically distinct conditions:

(i) **Existence:**

$$A_\Sigma \neq 0;$$

(ii) **Gauge admissibility:**

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0;$$

(iii) **Observable phase locking:** the internal phase data of Σ^2 must enter a coherent and temporally readable observable regime.

The third condition is not topological but dynamical and observational. A compactification surface may satisfy the geometric conditions for gauge activity yet remain observationally inert if its phase structure does not admit stable locking and globally comparable readout in the effective spacetime projection.

Remark Σ-3.10 (Two routes to darkness)**.** Dark sectors arise through two conceptually distinct mechanisms:

- **Topological darkness:**

$$c_1^{\mathrm{ind}}(\Sigma) = 0,$$

so the compactification is gauge-inactive at the structural level;

- **Observational darkness:**

$$c_1^{\mathrm{ind}}(\Sigma) \neq 0,$$

but coherent observable phase locking fails, so the sector is not dynamically readable in the effective projection.

4 Black Holes as Projection Failure

4.1 Black Holes in Six-Dimensional Ontology

In the FBT framework, four-dimensional spacetime is not fundamental. It emerges as a projected effective description of the underlying six-dimensional symplectic geometry under appropriate admissibility and regularity conditions.

Definition Σ-4.1 (Black hole as projection failure)**.** A black hole region is a domain

$$\mathcal{B} \subset \mathcal{M}^6$$

in which the projection

$$\pi : \mathcal{M}^6 \longrightarrow \mathcal{M}^4_{\text{obs}} \tag{8}$$

ceases to be a regular observable projection. Equivalently, the Jacobian degenerates or the effective four-dimensional spacetime description loses global validity.

Thus, within this ontology:

- black holes are not singular points or substances;
- they are regions where the four-dimensional observable description fails while the six-dimensional geometry remains regular.

4.2 Formation Mechanism: Breakdown of Observable Temporal Projection

Theorem Σ-4.2 (Breakdown of observable temporal projection)**.** *Let Σ^2 be a compactification surface with Berry curvature Ω_{phase}. There exists a finite threshold Ω_* such that whenever the local concentration of Ω_{phase} exceeds this threshold, no single observable projection can provide a globally consistent temporal ordering and coherent phase comparability for all admissible Berry loops on Σ^2.*

Proof sketch. As Ω_{phase} concentrates, distinct Berry loops can enter different Morse descent sectors, and their relative phases can no longer be organised by a single globally coherent observable ordering. Picard–Lefschetz theory then implies the loss of a unique dominant effective branch, so that a regular observable temporal projection ceases to exist. □

Corollary Σ-4.3 (Event horizon as visibility boundary)**.** *The event horizon is the geometric boundary beyond which no globally coherent observable temporal projection exists. It marks the boundary of observability rather than the boundary of existence.*

4.3 Interior Structure: Deep Dark Geometric Phase

Proposition Σ-4.4 (Black hole interior as deep dark phase)**.** *Within $\mathcal{B}$, the dominant compactification surfaces remain gravitationally active,*

$$A_\Sigma \neq 0,$$

while gauge activity and coherent observable temporal readout are suppressed. Hence the interior naturally realises an extreme dark geometric sector: present in the six-dimensional ontology, gravitationally active, but observationally inaccessible in the regular four-dimensional projection.

This implies:

- the black hole interior is not empty;
- it is not singular in the six-dimensional description;
- it corresponds to a high-density dark geometric phase of the same underlying structure that produces ordinary visible matter outside.

4.4 Resolution of the Singularity Problem

The singularities of classical general relativity arise from the attempt to extend a four-dimensional metric description into a regime where its geometric prerequisites fail.

Theorem Σ-4.5 (No true singularity)**.** *By Theorem Σ-2.6, symplectic rigidity and minimal area constraints forbid infinite compression of compactification surfaces. Therefore, no true geometric singularity occurs in* $\mathcal{M}^6$*.*

Corollary Σ-4.6. *General relativistic singularities are artefacts of four-dimensional overextension. The six-dimensional geometry remains regular and finite-capacity throughout.*

4.5 Hawking Radiation as Local Restoration of Observable Readout

Proposition Σ-4.7 (Hawking radiation as local restoration of observable readout)**.** *Near the event horizon, local regions of the compact phase sector may re-enter an admissible observable projection regime. Compactification surfaces then undergo transient transitions from observationally dark branches to temporarily readable branches. This process manifests effectively as Hawking radiation.*

Thus:

$$\boxed{\text{Hawking radiation is the local restoration of admissible observable readout.}}$$

5 Conclusion and Relation to Companion Papers

We have addressed three foundational questions within the FBT framework.

Symplectic rigidity as a unifying principle. Gromov's non–squeezing theorem provides a common geometric explanation for:

- the existence of a minimal action quantum and hence quantum uncertainty;
- the impossibility of true geometric singularities and hence gravitational regularity;
- the intrinsically nonlocal character of the symplectic ontology.

Geometric definition of visible and dark sectors. Matter sectors are classified by:

- gravitational activity through
$$A_\Sigma \neq 0;$$
- gauge admissibility through
$$c_1^{\mathrm{ind}}(\Sigma) \neq 0;$$
- observable visibility through admissible phase locking and coherent effective readout.

Dark sectors arise either from topological gauge inactivity or from the failure of coherent observable phase locking despite gravitational activity.

Black holes as projection failure. Black holes are not singularities of the underlying six-dimensional geometry, but regions where regular observable projection to four-dimensional spacetime breaks down. The event horizon is therefore a visibility boundary, while the underlying geometry remains regular.

Taken together, these results support a unified picture in which symplectic rigidity, compactification topology, and observable phase locking jointly determine the distinction between visible matter, dark sectors, and projection-failure regimes such as black holes.

6 Open Questions

Several directions remain for future investigation:

- **Quantification of observable phase locking.** The visibility condition introduced in Section 3 requires, beyond topological data, that the internal phase geometry of Σ^2 enter a coherent observable readout regime. Can this condition be quantified by a geometric invariant?

  Natural candidates include:

  - the integrated Berry phase over distinguished cycles of Σ^2, whose deviation from a reference quantised value might measure failure of coherent observable locking;
  - a Maslov-type index associated with admissible Lagrangian data in the compact phase sector, encoding the compatibility of internal phase transport with the effective temporal ordering of the observable projection;
  - an overlap or coherence functional between the induced phase connection on Σ^2 and the effective temporal ordering selected in the observable sector.

  A more refined possibility is that observability is controlled not by a binary condition (locked/unlocked), but by a continuous mismatch parameter
$$\epsilon,$$
  measuring the degree to which internal phase transport fails to enter a globally comparable observable regime. Perfect phase locking would correspond to a fully readable but dynamically frozen limit, whereas large mismatch would correspond

to observational darkness. The visible universe would then occupy an intermediate regime of near locking: sufficiently coherent to be observable, but sufficiently nontrivial to support evolution and structure formation.

- **Relation between $c_1^{\mathrm{ind}}(\Sigma)$ and intrinsic Berry curvature.** The induced first Chern class $c_1^{\mathrm{ind}}(\Sigma)$ is defined via projection of the Berry curvature on Σ^2. What is the precise relation between c_1^{ind} and the integral of the phase curvature over Σ^2? Does a nonzero induced class always require nontrivial topology in the embedding, or can it arise purely through projection structure?

- **Measure argument for dark matter dominance.** Theorem Σ-3.8 asserts that dark compactifications dominate by measure. Can this be made mathematically rigorous using techniques from symplectic topology, moduli theory, or pseudoholomorphic curve counting?

- **Black hole interior dynamics.** Section 4 characterised the black hole interior as a deep dark geometric phase where regular observable projection fails. Does this phase admit its own effective geometric description, and can it illuminate the microscopic origin of black hole entropy?

- **Observational signatures of observational darkness.** If some dark sectors arise not from topological triviality but from failure of coherent observable readout, could they leave distinct imprints in clustering, strong-gravity environments, or transitions near visibility boundaries?

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A Supplementary Material on Symplectic Rigidity

This appendix isolates the geometric mechanism underlying several apparently distinct phenomena discussed in the main text: quantum discreteness, the impossibility of infinite compression, and the absence of true spacetime singularities. All three arise from symplectic rigidity as manifested by Gromov's non–squeezing theorem.

Symplectic rigidity as geometric censorship

Let $(\mathcal{M}^6, \Omega)$ denote the fundamental six-dimensional symplectic manifold, equipped with compact symplectic surfaces

$$\Sigma^2 \hookrightarrow \mathcal{M}^6$$

with symplectic area

$$A_\Sigma := \int_{\Sigma^2} \Omega > 0, \qquad A_\Sigma \geq h. \tag{A.1}$$

A physically admissible neighbourhood supporting the compact phase sector must contain a transverse symplectic two-disk whose area is bounded below by h. Hence no sequence of admissible symplectic deformations can drive the corresponding phase-space cross-section to zero.

This yields a purely geometric obstruction to infinite compression: any attempted collapse of internal degrees of freedom to a point would require vanishing symplectic capacity, which is forbidden by symplectic rigidity once a positive compact phase area is present.

Non-squeezing as the geometric origin of quantum uncertainty

The same rigidity mechanism has a complementary quantum interpretation. The existence of a strictly positive lower bound h on the symplectic area of the compact phase sector implies that phase space cannot be partitioned into arbitrarily small cells. Under projection to the effective four-dimensional description, this minimal symplectic cell appears as a lower bound on simultaneous localisation of conjugate variables. Thus the uncertainty principle is interpreted as the shadow of symplectic non–squeezing in the full phase space.

Arnold-type existence and topological stability of matter states

A useful existence prototype for the slogan "matter is geometry" comes from the Arnold conjecture and Floer-theoretic refinements. On a compact symplectic surface, Hamiltonian evolution possesses fixed points or periodic orbits whose existence is controlled by topology. In the present interpretation, compact phase surfaces provide the minimal internal arena on which admissible matter-like response modes are stabilised. Observable matter states are then understood as equivalence classes of such invariant structures under admissible coarse-graining and projection.

Absence of local symplectic shape

Since all symplectic manifolds are locally Darboux-flat, no intrinsic local symplectic curvature or local symplectic shape exists. Hence the fundamental ontology cannot be local in the metric sense. Any local geometric content belongs to the effective projected description, not to the underlying symplectic carrier.

B Supplementary Material on Dark Matter Classification

This appendix records supplementary details on the classification of visible and dark sectors.

Unified geometric setup

Matter sectors arise from compactified symplectic surfaces

$$\Sigma^2 \hookrightarrow (\mathcal{M}^6, \Omega),$$

and are characterised primarily by:

(i) symplectic area

$$A_\Sigma := \int_{\Sigma^2} \Omega,$$

which controls gravitational activity;

(ii) induced first Chern class

$$c_1^{\mathrm{ind}}(\Sigma) \in H^2(\mathcal{M}^4_{\mathrm{obs}}, \mathbb{Z}),$$

which controls gauge admissibility.

Visible and dark sectors

A visible sector requires:

$$A_\Sigma \neq 0, \qquad c_1^{\mathrm{ind}}(\Sigma) \neq 0,$$

together with coherent observable phase locking.

A dark sector requires:

$$A_\Sigma \neq 0,$$

but fails at least one visibility condition, either through topological gauge inactivity

$$c_1^{\mathrm{ind}}(\Sigma) = 0$$

or through failure of coherent observable readout.

Measure dominance

The simultaneous satisfaction of nontrivial induced gauge topology and coherent observable phase locking is structurally selective. Hence dark compactifications, understood as those failing at least one of these conditions, dominate generically in the admissible moduli of compactifications.

Role of observable phase locking in visibility

Beyond topology and area, visibility requires that the internal phase data of the compactification enter a coherent observable regime. Topology determines what can couple in principle; observable phase locking determines what is actually readable in the effective temporal order. This yields a three-fold condition of visibility:

$$A_\Sigma \neq 0, \qquad c_1^{\mathrm{ind}}(\Sigma) \neq 0, \qquad \text{coherent observable phase locking.}$$

C Planck Constant from Integral Symplectic Geometry

This appendix provides the rigorous derivation of the geometric identification

$$h = \min\left\{ \int_{\Sigma^2} \Omega > 0 \right\}$$

used in the main text. The derivation proceeds through quantum-phase consistency, prequantisation integrality, admissible lattice discreteness, and stability under non–squeezing.

C.1 Quantum-phase principle

Assumption Σ-C.1 (Quantum-phase principle)**.** Any admissible history contributes to the path integral with a phase

$$\exp(iS/\hbar),$$

and only the phase modulo 2π is observable.

In the present setting, admissible compactification sectors are encoded by compactified two-cycles $\Sigma^2 \hookrightarrow \mathcal{M}^6$ with action

$$S(\Sigma^2) = \int_{\Sigma^2} \Omega. \tag{C.1}$$

Definition Σ-C.2 (Universal action quantum)**.** A positive constant h is called a universal action quantum if

$$\int_{\Sigma^2} \Omega \in h\,\mathbb{Z}$$

for every admissible compactification cycle. We set

$$\hbar := \frac{h}{2\pi}.$$

C.2 Prequantisation

Theorem Σ-C.3 (Prequantisation integrality)**.** *There exists a Hermitian line bundle $L \to \mathcal{M}^6$ with unitary connection ∇ satisfying*

$$F_\nabla = \frac{1}{\hbar}\Omega$$

if and only if

$$\frac{1}{2\pi\hbar}[\Omega] \in H^2(\mathcal{M}^6, \mathbb{Z}),$$

equivalently,

$$\int_{\Sigma^2} \Omega \in h\,\mathbb{Z}$$

for all admissible integral two-cycles.

Proof. This is the classical prequantisation theorem. □

C.3 Structural minimality

Assumption Σ-C.4 (Admissible compactification lattice)**.** The set of physically admissible compactification classes

$$\mathcal{H}_2^{\mathrm{adm}} \subset H_2(\mathcal{M}^6, \mathbb{Z})$$

is nontrivial and generated by finitely many primitive classes. Equivalently, the set of admissible symplectic periods forms a discrete subgroup of $(\mathbb{R}, +)$.

Proposition Σ-C.5 (Existence of a minimal nonzero symplectic period)**.** *Under Assumptions Σ-C.1 and Σ-C.4, there exists a unique minimal nonzero value $A_{\min} > 0$ such that every admissible symplectic period is an integer multiple of $A_{\min}$:*

$$\int_{\Sigma^2} \Omega \in A_{\min}\,\mathbb{Z}, \qquad A_{\min} = \min\left\{ \int_{\Sigma^2} \Omega > 0 : \ [\Sigma^2] \in \mathcal{H}_2^{\mathrm{adm}} \right\}. \tag{C.2}$$

Proof. A discrete subgroup of $(\mathbb{R}, +)$ is of the form $\alpha\mathbb{Z}$. Nontriviality implies $\alpha > 0$. □

Definition Σ-C.6 (Geometric identification of h)**.** We define

$$h := A_{\min}.$$

C.4 Role of non–squeezing

Non–squeezing does not determine the numerical value of h. Its role is to ensure that, once a minimal symplectic period exists, it is stable under admissible continuous symplectic deformations.

C.5 Summary

$$\boxed{h = \min\left\{ \int_{\Sigma^2} \Omega > 0 : \ [\Sigma^2] \in \mathcal{H}_2^{\mathrm{adm}} \right\}, \qquad \frac{1}{2\pi\hbar}[\Omega] \in H^2(\mathcal{M}^6, \mathbb{Z}).}$$

Thus the Planck constant is identified with the minimal nonzero symplectic period on admissible compactification cycles, while symplectic rigidity explains the stability of this quantum under admissible deformations.